Math 10B with Professor Stankova
Quiz 4; Tuesday, 2/19/2019
Section #203; Time: 11 AM
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Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True **FALSE** If we want to show that the statements S_n are true for all $n \geq 0$, we need to prove the base case $n = 1$.

   Solution: The base case is $n = 0$.

2. **TRUE** False If we use induction to prove a solution to $a_n = na_{n-1} + 3a_{n-2} - a_{n-3}^2$, then we will need to use S_n, S_{n-1}, and S_{n-2} to prove S_{n+1}.

   Solution: This is a third-order relation so we will need to assume the IH for the past 3 values of n to prove $n + 1$.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (7 points) Prove that $1 - 2 + \cdots + (-2)^n = \frac{1-(-2)^{n+1}}{3}$ for all $n \geq 0$.

   Solution: First we prove the base case $n = 0$. Then the LHS is 1 and the RHS is $\frac{1-(-2)}{3} = 1 =$LHS as required.
   Now assume the inductive hypothesis IH: $1 - 2 + \cdots + (-2)^n = \frac{1-(-2)^{n+1}}{3}$ for some $n \geq 1$.
   Now we want to prove that $1 - 2 + \cdots + (-2)^{n+1} = \frac{1-(-2)^{n+2}}{3}$. We have that the left hand side is

$$\begin{aligned} LHS &= 1 - 2 + \cdots + (-2)^n + (-2)^{n+1} \\ &\overset{IH}{=} \frac{1-(-2)^{n+1}}{3} + \frac{3(-2)^{n+1}}{3} \\ &= \frac{1+2(-2)^{n+1}}{3} \\ &= \frac{1-(-2)(-2)^{n+1}}{3} = \frac{1-(-2)^{n+2}}{3} = RHS \end{aligned}$$

   Thus, by MMI, we know that $1 - 2 + \cdots + (-2)^n = \frac{1-(-2)^{n+1}}{3}$ for all $n \geq 1$.

(b) (3 points) What is the probability that when picking a hand of 5 cards out of a deck of 52 cards, you don't have any pairs (or triples/four of a kind)?

Solution: The total number of ways to pick 5 cards is $\binom{52}{5}$. Then, since we don't have any pairs, we have 5 different values and there are $\binom{13}{5}$ ways to choose them. Then, for each value, there are 4 suits so the probability is

$$\frac{\binom{13}{5}4^5}{\binom{52}{5}}.$$